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Abstract: In this work, the spatial and temporal distributions of small thermal and electromagnetic perturbations in a plane semi-infinite superconducting sample are studied. Based on a system of equations for temperature, magnetic induction, and vortex motion, a dispersion relation was obtained that determines the growth (or decay) increment of small perturbations. It was shown that, under certain conditions, depending on the values of the parameters of the system, flux jumps of the magnetic flux is observed. On the other hand, the phenomenon of magnetic flux oscillations - the oscillation of vortex matter as a result of thermomagnetic instability of the critical state in a superconductor is theoretically investigated. The spatial and temporal distributions of small thermal and electromagnetic perturbations in a plane semi-infinite superconducting sample are studied. Based on the system of equations for temperature, magnetic induction, and vortex motion, a dispersion relation was obtained that determines the growth (or decay) increment of small perturbations. It was shown that, under certain conditions, depending on the values of the parameters of the system, jumps - oscillations of the magnetic flux can be observed.

Key words: *superconductors, small perturbations, flux jumps, vortex, critical state.*

INTRODUCTION

The phenomenon of magnetic flux jumps as a result of thermo magnetic instability of the critical state in a superconductor is theoretically investigated [1]. The spatial and temporal distributions of small thermal and electromagnetic perturbations in a plane semi-infinite superconducting sample are studied. Based on the system of equations for temperature, magnetic induction, and vortex motion, a dispersion relation was obtained that determines the growth (or decay) increment of small perturbations. It was shown that, under certain conditions, depending on the values of the parameters of the system, flux jumps of the magnetic flux can be observed.

Since the discovery of this phenomenon in the 1960s, thermomagnetic instability caused by flux jump has been studied in superconducting slabs, bulk, and films. Based on the critical state model, a theory has been proposed to explain the flux jump of an infinite superconducting slab subjected to an external magnetic field. The total magnetic flux in the superconductor changes with the increase in the applied field. This will induce a dissipation of heat, and thus, the shielding ability of the magnetic field is reduced in the superconductor. The reduction of shielding ability will also lead to the motion of magnetic flux and more heat. This positive feedback can trigger thermomagnetic instability, further causing flux jump. The flux jump and flux avalanche can lead to the abrupt rise of temperature in the superconducting bulk and film, which has also been verified in MgB_2 by experiment. Generally speaking, flux jump and flux avalanches must be avoided during safe operation of the bulk in order to decrease the likelihood of destruction. The flux jump or flux avalanche has been reported by many researchers. The flux jump is accompanied by the temperature rise, and the temperature may be much larger than the critical temperature. Thus, it is important to consider the mechanical behavior of bulk superconductors during the flux jump.

Thus, the flux jump can lead to the degradation of performance, which reduces thermal stability and seriously threatens the safe operation of the bulk superconductor. Furthermore, the temperature and electromagnetic field can both have a rapid change during the flux jump, which can generate large thermal stress and Lorentz force. Moreover, the large electromagnetic bodyforce is also able to induce large mechanical deformation in the superconductor. It was reported that the thermal stress and electromagnetic stress may result in the fracture of the bulk superconductor. Thus, it is important to consider the mechanical behavior of bulk superconductors during the flux jump.

BASIC EQUATIONS

The distribution of magnetic induction, electric field, and transport current in the superconductor are determined by the following equation

$$\mathrm{rot}\,\vec{B} = m_0\vec{j}\,. \qquad (1)$$

$$\mathrm{rot}\,\vec{E} = \frac{d\vec{B}}{dt}\,. \qquad (2)$$

Accordingly, the temperature distribution in the sample is determined by the heat conduction equation

$$\nu(T)\frac{dT}{dt} = \nabla[\kappa(T)\nabla T] + \vec{j}\vec{E}\,, \qquad (3)$$

where ν and κ are the coefficients of heat capacity and thermal conductivity of the sample, respectively. Addiction $j = j_C(T,B,E)$ is determined by the following critical state equation

$$j = j_C(T,B) + j(E)\,.$$

We will use the Bean model $j_C = j_C(B_e,T) = j_0 - a(T_C - T_0)$, where B_e is the value of the external magnetic induction; $a = \frac{j_0}{T_C - T_0}$; j_0 - equilibrium current density, T_0 and T_C - initial and critical temperature of the sample, respectively [1]. In the flow creep mode, the current-voltage characteristic of superconductors is nonlinear, due to the heat-activated motion of vortices [2]. The dependence $j(E)$ in the flow creep mode is described by the expression [3]

$$j = j_C \left[\frac{E}{E_0} \right]^{1/n} , \qquad (4)$$

where E_0 is the value of the electric field strength at $j = j_C$; the constant parameter n depends on the pinning mechanisms. In the case when n =1, relation (4) describes a viscous flow [1]. For sufficiently large values of n , the last equality defines Bean's critical state $j \propto j_c$. When 1< n <∞, relation (4) describes the nonlinear creep of the flow [4]. In this case, the differential conductivity is determined by the equality

$$\sigma = \frac{d\vec{j}}{d\vec{E}} = \frac{j_C}{nE_B} . \qquad (5)$$

According to equation (5), the differential conductivity increases with increasing background electric field E_B and essentially depends on the value of the rate of change of magnetic induction according to the equality $E_B \propto \dot{B}_E x$. Let's formulate the basic equations describing the dynamics of the development of thermal and electromagnetic disturbances for a simple case - a superconducting flat semi-infinite sample (x >0)

$$\nu \frac{d\Theta}{dt} = \kappa \frac{d^2\Theta}{dx^2} + j_c \varepsilon , \qquad (6)$$

$$\frac{d^2\varepsilon}{dx^2} = \mu_0 \left[\frac{j_c}{nE} \frac{d\varepsilon}{dt} - \frac{dj_c}{dT} \frac{d\Theta}{dt} \right] . \qquad (7)$$

We represent the solution of system (6), (7) in the form

$$dT(x,t) = (T_c - T_0) Q(z) e^{\frac{\gamma t}{t_0}} , \qquad (8)$$

$$dE(x,t) = E_c \varepsilon(z) e^{\frac{gt}{t_0}} . \qquad (9)$$

where γ is the eigenvalue problem to be determined. It can be seen from the last system of equations that the characteristic time for the development of thermal and electromagnetic perturbations of the order of - t_0/γ [5]. We have introduced the following dimensionless parameters and variables

$$\beta = \frac{\mu j_c^2 L^2}{\nu(T_c - T_0)}, \qquad t_0 = \frac{\mu j_c L^2}{E_c},\ z = \frac{x}{L}, \qquad \tau = \frac{t}{t_0},\ .\ e = \frac{\delta E}{E_c},\ t_0 = \frac{\sigma\nu(T_c - T_0)}{j_c^2},\ l = \frac{\nu(T_c - T_0)}{\mu_0 j_c^2}$$

$$\gamma = \frac{1-n}{n}.$$

Let’s consider the problem within the adiabatic approximation, when $\tau << 1$, i.e. [5], the diffusion of the magnetic flux occurs faster than the thermal diffusion. Then, we obtain the following equation in the quasi-stationary approximation

$$\frac{d^2Q}{dz^2} - zQ = 0. \qquad (10)$$

Since, when deriving the last equation, we neglected thermal effects, only the electrodynamic boundary should be put in (10)

$$Q(1,t) = 0, \qquad \frac{dQ(0,t)}{dt} = 0. \qquad (11)$$

The stability criterion of the magnetic flux jumps is determined by the values of $\mathrm{Re}\,\gamma \le 0$. Then, using the second boundary condition $Q(1) = 0$, we obtain the following equation for determining the parameter γ

$$J_{2/3}(a_n) = J_{-2/3}(a_n)$$

A nontrivial solution of the last equation, taking into account the boundary conditions (10), exists only for certain values

$$a_1 = \rho^{2/3}\gamma\ .$$

where a_1 are the roots of the characteristic Bessel function. After simple transformations, we obtain the following stability criterion for the flux jumps

$$B_c = \frac{4\pi j_c}{c}\sqrt{\frac{\kappa(T_c - T_0)}{j_c n \dot{B}_e}}\ . \qquad (12)$$

It is easy to see that the threshold value of B_c flux jump stability mainly depends on the type of background electric field initiated by a change in external magnetic induction $E_b \approx \dot{B}_e$ [6]. The value of B_c decreases monotonically with increasing of the external magnetic field induction rate in the sample.

2. The oscillation of vortex matter as a result of thermomagnetic instability

Recently, great attention has been paid to the phenomenon of magnetic flux oscillations arising as a result of thermomagnetic instability in superconductors [7]. In the process of studying the dynamics of thermomagnetic instabilities, vibrational modes in the mixed state of a superconducting Nb-Ti sample were detected as a result of a catastrophic avalanche [8]. To explain the observed oscillation processes, a theoretical model was proposed that takes into account the inertial properties of vortex matter. These oscillation phenomena were interpreted as the result of the existence of a finite value of the effective mass of the vortex, i.e. oscillations can be considered as a manifestation of the inertial properties of vortex matter [9].

We consider the phenomenon of magnetic flux oscillations - the oscillation of vortex matter as a result of thermomagnetic instability of the critical state in a superconductor is theoretically investigated. The spatial and temporal distributions of small thermal and electromagnetic perturbations in a plane semi-infinite superconducting sample are studied. Based on the system of equations for temperature, magnetic induction, and vortex motion, a dispersion relation was obtained that determines the growth (or decay) increment of small perturbations. It was shown that, under certain conditions, depending on the values of the parameters of the system, jumps - oscillations of the magnetic flux can be observed.

The system of equations of macroscopic electrodynamics is used to simulate the evolution of temperature and electromagnetic field perturbations. The distribution of magnetic induction $\vec{B}(r, t)$ and transport current $\vec{j}(r, t)$ in a superconductor is given by the equation

$$\mathrm{rot}\,\vec{B}=\frac{4\pi}{c}\vec{j}\ . \qquad (13)$$

The relationship between the magnetic induction $\vec{B}(r, t)$ and electric field $\vec{E}(r, t)$ is described by Maxwell's equations

$$\mathrm{rot}\,\vec{E}=-\frac{1}{c}\frac{d\vec{B}}{dt}\ , \qquad (14)$$

$$\vec{E}=\frac{v}{c}\vec{B}\ . \qquad (15)$$

The equation of motion of the vortices can be written in the form

$$m\frac{dV}{dt}+\eta V+F_L+F_P=0\,, \qquad (16)$$

where m is the mass of the vortex of unit length, $F_L=\frac{1}{c}\vec{j}\,\vec{\Phi}_0$ is the Lorentz force, $F_L=\frac{1}{c}\vec{j}_c\,\Phi_0$ is the pinning force, $\eta=\frac{\vec{\Phi}_0 H_{C2}}{c^2\rho_n}$ is the viscosity coefficient, ρ_n is the resistance in the normal state, $\vec{\Phi}_0=\frac{\pi hc}{2e}$ is the magnetic flux quantum, is the upper critical field [4]. In combining the relation (14) with Maxwell's equations, we obtain a nonlinear diffusion equation for the magnetic flux induction $\vec{B}(r, t)$ in the following form

$$\frac{d\vec{B}}{dt}=\nabla\left(\vec{v}\cdot\vec{B}\right) \qquad (17)$$

$$m\frac{dv}{dt}+\eta v=-\frac{1}{c}\Phi_0(j\text{-}j_c). \qquad (18)$$

The temperature distribution in superconductor is governed by the heat conduction diffusion equation

$$\nu\frac{dT}{dt}=\Delta\left[\kappa(T)\Delta T\right]+\vec{j}\vec{E}, \qquad (19)$$

where $\nu=\nu(T)$ and $\kappa=\kappa(T)$ are the heat capacity and thermal conductivity coefficients of the sample, respectively. We use the Bean model for the current density $\vec{j}(T, E, B)$ and assume that it does not depend on the magnetic field induction, $j=j_C(B_e, T)$, i.e., $j_C=j_0-a\left(T\text{-}T_0\right)$ [5], where B_e is the value of the external magnetic induction; $a=j_0/\left(T_C\text{-}T_0\right)$; j_0 is the equilibrium current density, T_0 and T_C are the initial and critical temperature of the sample, respectively [6]. We assume that the external magnetic field $\vec{B}=(0, 0, B_e)$ is directed along the z axis and the magnetic field sweep rate $\dot{B}_e$=const is constant.

RESULTS AND DISCUSSIONS

Let us present a solution of equations (14)-(16) in the form

$$B=B_e+b(x,t),$$

$$v=v_0+v(x,t), \qquad (20)$$

$$T=T_0+\Theta(x,t),$$

where $T_0(x)$, $B_e(x)$ and $v_0(x)$ are the solutions to the unperturbed equations, which can be obtained within a quasi-stationary approximation. Substituting the above solution (18) into equations (14)-(17) we obtain the following system of differential equations

$$\frac{d\Theta}{dt}=2\upsilon-\beta\Theta, \qquad (21)$$

$$\mu\frac{d\upsilon}{dt}+\upsilon=-\frac{db}{dx}+\beta\Theta, \qquad (22)$$

$$\frac{db}{dt}=\left(\frac{db}{dx}+b\right)+\left(\frac{d\upsilon}{dx}+\upsilon\right). \qquad (23)$$

where dimensionless parameters $\mu=\frac{c\vec{\Phi}_0}{4\pi\eta^2}\frac{B_e}{2L^2}$ and variables $b=\frac{B}{B_e}=\frac{c}{4\pi}\frac{B}{j_C L}$, $\Theta=\frac{4\pi}{c}\frac{2\nu}{B_e^2}$, $v=V\frac{t_0}{L}$, $z=\frac{x}{L}$, $\tau=\frac{t}{t_0}=\frac{c\vec{\Phi}_0}{4\pi\eta}\frac{B_e}{2\mu_0 j_C L^2}$ were introduced. Here $L=\frac{c}{4\pi}\frac{B_e}{j_C}$ is the depth of penetration of the magnetic field into the superconductor [7].

We assume that the small thermal and magnetic perturbations has $\Theta(x,t), b(x,t), v(x,t)\cdot\exp(\gamma t)$, (where γ is the eigenvalue of the problem to be determined), we obtain from the system Eqs. (21)-(23) the following dispersion relations to determine the eigenvalue problem

$$\frac{d^2b}{dx^2}-\left[(\gamma+\beta)\mu-2\beta\right]\frac{db}{dx}+\left[(\mu+1)\gamma^2+(\mu-1)\beta-(\mu-1)\beta\right]b=0 \qquad (24)$$

The instability of the magnetic front, as a rule [7-9], is determined by the positive values of the increment $\mathrm{Re}\,\gamma \geq 0$. Then we can assume that the instability arises under the condition $\mathrm{Re}\,\gamma=0$. Analysis of the dispersion relation shows that the growth increment is positive $\mathrm{Re}\,\gamma \geq 0$ if the condition $\mu \geq \mu_C = 2$ is met. In this case $\mu \geq \mu_C$, the small perturbations increase with time and the magnetic flux front is unstable. In the case when the increment is negative $\mu \geq \mu_C$ and any small perturbation will decay. At a critical value, the increment is zero $\gamma=0$ [8].

In the particular case when $\mu = 1$, the increment γ is determined by the stability parameter β. Then, the stability criterion can be represented as β>1. In another particular case, when the thermal effects are insignificant (β=1), the following dispersion relation can be obtained

$$\frac{d^2b}{dx^2} - \mu\frac{db}{dx} + \left[(\gamma\text{-}1)(\mu+1)\right]b=0 \qquad (25)$$

Representing the solution of the dispersion equation (25) in the form

$$b \sim e^{-ikx}$$

we can obtain the dependence of the increment γ on the wave vector k. An analysis shows [7] that when k<kc=μ, the increment is positive and a small perturbation increases with time. For the values of the wave vector $k>k_c$, the quantity γ is negative and the small perturbation decays exponentially. It can be shown [9], that for $k = k_c$ the increment is $\gamma = 0$. If the wave vector tends to zero $k \to 0$ or infinity $k \to \infty$, the quantity $\gamma = 1$ and a small perturbation increases. In this case, the quantity γ is determined by the relation

$$\gamma = \frac{2\mu}{\mu+1}.$$

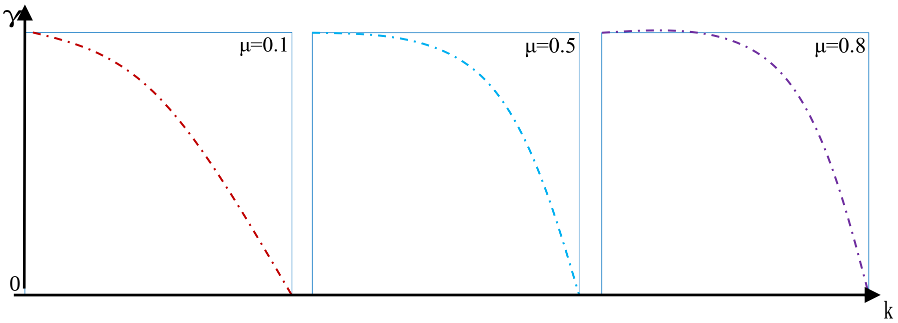

Fig 1. The dependence of the growth rate of γ on the wave vector k for μ=0.1, 0.5, 0.8.

For μ = 0, the value of the increment is γ = 0. For μ = 1, the value of γ = 1. The dependence of the growth rate of γ on the wave vector is shown in Fig. 1. for various values of the parameter μ. As μ increases, the parameter γ increases. At certain values of the parameter μ, magnetic flux jumps are observed, which take into account the inertial properties of the vortices.

CONCLUSION

Thus, based on a system of equations for temperature, magnetic induction, and vortex motion, a dispersion relation was obtained that determines the growth (or decay) increment of small perturbations. It was shown that, under certain conditions, depending on the values of the parameters of the system, flux jumps of the magnetic flux are observed. The phenomenon of oscillation of the vortex matter as a result of thermomagnetic instability in a superconductor is theoretically investigated. The spatial and temporal distributions of small thermal and electromagnetic perturbations in a plane semi-infinite superconducting sample are studied. Based on the system of equations for temperature, magnetic induction, and vortex motion, a dispersion relation was obtained that determines the growth (or decay) increment of small perturbations. It was shown that, under certain conditions, depending on the values of the parameters of the system, jumps - oscillations of the magnetic flux is observed.

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